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***Abstract*—We present a compressed sensing framework for quantitative MRI based on Magnetic Resonance Fingerprinting. We show that, as long as the excitation sequence induces persistent excitation, we are able to achieve accurate recovery of the proton density, T_1, T_2 and off-resonance maps simultaneously from very short pulse sequences.**

I. Introduction

A new type of MRI acquisition scheme called Magnetic Resonance Fingerprinting (MRF) [3] offers full quantification of multiple tissue properties simultaneously through a single acquisition process. The procedure is composed of 4 steps: (1) the material magnetization is excited through a sequence of random RF pulses; (2) each pulse response is recorded over a small portion of k-space; (3) a sequence of highly aliased magnetization response images are formed using back projection; and (4) parameter maps (proton density, ρ, T_1, T_2 and off-resonance, δf) are formed using a bank of matched filters, applied voxelwise.

We investigate this idea from a compressed sensing (CS) perspective and leverage recent results from [1] and develop a recovery algorithm with good theoretical guarantees.

II. The Bloch response manifold

The MRF process is based upon a random pulse excitation sequence. Denote the magnetization response image sequence by $X \in \mathbb{C}^{N\times L}$, with $X_{i,t}$ denoting the magnetization for voxel i at the t^{th} readout. The magnetization response at any voxel can be written as a parametric nonlinear mapping from $\{\rho_i, \theta_i\}$ to the sequence $X_{i,:}$ as:

$$X_{i,:} = \rho_i B(\theta_i; \alpha, \mathrm{TR}) \in \mathbb{C}^{1\times L}, \tag{1}$$

where L is the sequence length, $\theta_i = \{T_1, T_2, \delta f\} \in \mathcal{M}$ is the set of unknown parameters and $B : \mathcal{M} \to \mathbb{C}^{1\times L}$ is a smooth mapping induced by the Bloch dynamics.

Inferring $\{\rho_i, \theta_i\}$ from an estimate for $X_{i,:}$ (assuming identifiability) can be done by projecting onto a discretization of the cone of the response manifold, which we denote as $\mathbb{R}_+\mathcal{B}$.

Let $\theta_i^{(k)} = \{T_1^{(k)}, T_2^{(k)}, \delta f^{(k)}\}_{k=1:P}$ be a discrete sampling of $\mathcal{M}$ and define the MRF "dictionary" $D \in \mathbb{C}^{P\times L}$ of the magnetization responses as: $D_k = B(\theta_i^{(k)}; \alpha, \mathrm{TR})$, $k = 1, \dots, P$. The projection is given by the maximum matched filter of the voxel response sequence with the elements of D. After which the Bloch parameters can be retrieved using a look up table.

III. k-space Sampling

Unfortunately, it is impractical to observe the full spatial magnetization $X_{:,t}$ at each readout within the necessary time window and we must resort to some form of undersampling in k-space, which we denote by the mapping: $Y = h(X)$.

In order to ensure parameter map recovery, we now exploit tools from compressed sensing. In particular we would like h to induce a low distortion embedding of the cone of the product response manifold, $(\mathbb{R}_+\mathcal{B})^N$, or equivalently satisfy a suitable RIP. In order to achieve this it is useful to characterize the persistent discrimination of the different magnetization responses. We quantify this persistence through the *flatness* of the *chords* of $\mathbb{R}_+\mathcal{B}$.

Definition 1: Let U be a collection of vectors $\{u\}$ in $\mathbb{C}^L$. We denote the *flatness* of the these vectors by

$$\lambda := \max_{u\in U} \|u\|_\infty / \|u\|_2. \tag{2}$$

Note that from standard norm inequalities $L^{-1/2} \leq \lambda \leq 1$.

For our sampling operator we consider regularly subsampling k-space by a factor of p in one direction with random shifts at each readout time. This can be achieved using a randomized version of multishot *Echo-Planar Imaging* (EPI). The following theorem shows that random EPI can provide the desired RIP.

Theorem 1 (RIP for random EPI): Given an excitation response cone $\mathbb{R}_+\mathcal{B}$ of dimension $d_{\mathcal{B}}$, whose chords have a flatness λ, and a random EPI operator $h : (\mathbb{R}_+\mathcal{B})^N \to \mathbb{C}^{M\times L}$. With probability at least $1-\eta$, h is a restricted isometry on $(\mathbb{R}_+\mathcal{B})^N - (\mathbb{R}_+\mathcal{B})^N$ with constant δ as long as

$$\lambda^{-2} \geq C\delta^{-2} p^2 d_{\mathcal{B}} \log(N/\delta\eta), \tag{3}$$

for some constant C independent of $p, N, d_{\mathcal{B}}, \delta$ and η.
See [2] for further details.

IV. Compressed Quantitative MRI

Assuming that $Y = h(X)$ has a suitable RIP we can retrieve $\{\rho, \theta\}$ from Y using an efficient iterated projection algorithm [1] along with our discretized Bloch response model.

$$X^{(n+1)} = \mathcal{P}_{(\mathbb{R}_+\mathcal{B})^N}\left[X^{(n)} + \mu\, h^H\Big(Y - h\Big(X^{(n)}\Big)\Big)\right], \tag{4}$$

where n is the recursion index, $\mathcal{P}_{(\mathbb{R}_+\mathcal{B})^N}$ is the projection onto the signal model $(\mathbb{R}_+\mathcal{B})^N$ approximated using D, and μ is a stepsize, which we select adaptively. We call the resulting algorithm BLIP (BLoch response recovery via Iterated Projection).

V. Simulations

In simulations on an anatomical brain phantom the BLIP procedure was able to achieve near oracle performance with a pulse sequence length of ~ 200, substantially shorter than the already impressive MRF performance. For full details see [2].

References

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